Comprehensive Calculations on the OZI-forbidden Nonleptonic Decays of Orthoquarkonia $J/\psi(\Upsilon) \to \pi\pi, \rho\pi$

Tong Li[1], Shu-Min Zhao[1,2] and Xue-Qian Li[1]

[1] *Department of Physics, Nankai University, Tianjin 300071, China*
[2] *Department of Physics and Technology, Hebei University, Baoding 071002, China*

Abstract:
In this work, we calculate the decay rates of the OZI-forbidden processes $J/\psi(\Upsilon) \to \pi\pi, \rho\pi$ at the order of the leading-twist distribution amplitude. The process of $J/\psi(\Upsilon) \to \pi^+\pi^-$ violates isospin conservation and the amplitude is explicitly proportional to the isospin violation factor $m_u - m_d$, our numerical results on their decay rates are consistent with the data. The process $J/\psi(\Upsilon) \to \rho\pi$ violates the hadronic helicity conservation and should be suppressed, as indicated in literature, its decay rate can only be proportional to m_q^2 at the order of leading twist. Our theoretical evaluation confirms this statement that the theoretical evaluation on $\Gamma(J/\psi(\Upsilon) \to \rho\pi)$ is almost one order smaller than the data unless the model parameters take certain extreme values. It may imply that the sizable branching ratio of $J/\psi(\Upsilon) \to \rho\pi$ should be explained by either higher twist contributions or other mechanisms.

I. INTRODUCTION

It is generally believed that the narrowness of the ground states of heavy quarkonia J/ψ and Υ is due to the so called OZI suppression[1]. This OZI rule demands that if there are no quark lines connecting the initial and final hadron states, the processes are suppressed. At beginning, it seemed to be a phenomenological principle, however, further studies indicate that the suppression may originate from the loop suppression which can be precisely evaluated in the framework of perturbative QCD. More than 20 years ago, the OZI-suppressed radiative decays of orthoquarkonia was investigated by Körner et al. in perturbative QCD[2], where reasonable approximations were adopted. Since then, technique for calculating loop diagrams has been greatly improved and knowledge on the wavefunctions of light mesons is much enriched. Meanwhile more data have been accumulated and the corresponding experimental measurements become more precise[3, 4], all the experimental progress indeed provides us with a possibility to test our theoretical framework where the perturbative and non-perturbative effects are factorized and a convolution integral over them results in the physical transition amplitude. Following their work, we have also re-calculated the rates of $J/\psi(\Upsilon) \to \gamma + \pi^0(\eta,\ \eta')$ which are respectively isospin-violated, flavor-SU(3)-violated and flavor-SU(3)-favored processes without any approximations at one-loop level[5].

In fact, there may exist other possible mechanisms which also contribute to the concerned processes of $J/\psi(\Upsilon) \to PP$ and VP where P and V stand for pseudoscalar and vector mesons respectively[6, 7, 8], therefore to fully understand such reactions, a complete calculation on the OZI-suppressed non-leptonic decay processes is obviously necessary and should be possible with our present knowledge. Comparing with the radiative decays, theoretical evaluation of the rate of the non-leptonic decays is much more complicated. In the radiative decays, a photon is emitted as a free particle escaping away from the reaction and it does not participate in strong interaction. For the non-leptonic decay, the two (at least) daughter hadrons tangle together by exchanging gluons, therefore one not only needs to carry out the complicated Feynman integrations of four-point and five-point loop functions (i.e. D- and E-functions), but also there are more Feynman diagrams than the radiative decays. In this work we obtain the transition amplitude by carefully calculating the loop integrations. Following the standard procedure[9], one can reduce the 5-point loop functions into 4-point and 3-point loop functions which are then evaluated in terms of the program "LoopTools"[10, 11]. Moreover, one needs to carefully handle the color factors whereas they are much simpler in the radiative decays.

In this work, we are going to make a full calculation on the OZI-suppressed processes of $J/\psi(\Upsilon) \to \pi\pi$ and $J/\psi(\Upsilon) \to \rho\pi$ at the order of leading twist.

The reason to only consider $\pi^{\pm,0}$ and $\rho^{\pm,0}$ as the produced pseudoscalar and vector mesons is following. The processes are non-leptonic decays, at least three hadrons are involved and to theoretically evaluate the rates, one not only needs to calculate the complicated loop integrations at quark-gluon level, but also have to deal with the hadronic matrix elements which are fully governed by the non-perturbative QCD effects. However, at present, a completely reliable way to calculate the non-perturbative QCD effects is lacking, so that some phenomenological models must be invoked. In the decays of $J/\psi(\Upsilon) \to \pi\pi, \rho\pi$, the product mesons are light and can be nicely described in terms of the light-cone distribution amplitudes. Since π and ρ are composed of only u, d and $\bar{u}, \bar{d}$ whose masses are approximately equal, due to the obvious symmetry, the distribution functions are more symmetric and reliable, at least for the leading twist order. By contraries, for the distribution functions of $K(K^*)$, η and η', the produced mesons are composed of constituents $u(d)$ and s quarks which have a large difference in mass, thus one would expect larger uncertainties in the evaluation of hadronic matrix elements. Therefore, in this work, these final states are not concerned.

A simple analysis indicates that $J/\psi(\Upsilon) \to \pi\pi$ is an isospin-violating process. Namely the pions are treated as identical particles once the isospin symmetry is adopted in the analysis. Concretely, by the conservation of angular momentum, the two pions are in the p-wave state, since pions are identical bosons, the wave function of the two-pion system must be totally symmetric, so that the isospin of the system should be 1 as

$$\frac{1}{\sqrt{2}}(|1,1\rangle|1,-1\rangle - |1,-1\rangle|1,1\rangle) \equiv \frac{1}{\sqrt{2}}(|\pi^+\rangle|\pi^-\rangle - |\pi^-\rangle|\pi^+\rangle).$$

That requires that the process of $J/\psi \to \pi^0\pi^0$ is strictly forbidden. The isospin violation effects are expressed in the mass difference of u and d quarks which appears at the loop calculations, and the factor $m_u - m_d$ will be explicitly shown in the expressions derived at the quark level. Even though, we only consider the leading twist contribution of the light-cone wave functions which are independent of the quark masses, we still count in the mass splitting which results in the isospin violation. Turn to the processes $J/\psi(\Upsilon) \to \rho\pi$, in contrast with the $\pi\pi$ case, ρ and π are not identical particles, therefore the anti-symmetry requirement which enforces the two-pion system to be in isospin 1 state, is dismissed, so that the $\rho\pi$ system can be in isospin 0 state and it guarantees the iso-spin conservation for the decay process $J/\psi(\Upsilon) \to \rho^0\pi^0$. This observation seems to demand that the branching ratio of $J/\psi(\Upsilon) \to \rho\pi$ should be much larger than the isospin violating process $J/\psi(\Upsilon) \to \pi\pi$ and the data indeed support this statement. Moreover, in $J/\psi(\Upsilon) \to \rho\pi$, the isospin 0 state of $\rho\pi$ is dominant, so that the branching ratios of $J/\psi(\Upsilon) \to \rho^0\pi^0$ and $J/\psi(\Upsilon) \to \rho^+\pi^- + \rho^-\pi^+$ roughly retain a relation of 1:2. However, as indicated in Refs.[12, 13], such processes

violate the hadronic helicity conservation because gluons and photon do not carry hadronic helicities. A non-zero theoretical prediction on the rate at the order of leading twist must come from a violation of the hadronic helicity conservation. It is indicated that such a violation is proportional to the light quark mass, therefore one can expect that the directly calculated OZI suppressed amplitude should be proportional to $m_q^2(q = u, d)$. Our calculation confirms this mechanism (see the text for details).

The data[14] tell us an opposite conclusion that the helicity-violated process $J/\psi(\Upsilon) \to \rho\pi$ has sizable branching ratio and almost is one of the dominant modes in J/ψ and Υ decays. The discrepancy should be explained, some suggestions that the next-to-leading twist contribution, higher Fock states and other mechanisms such as the hadronic loop and glueball intermediate states etc. are taken into account, are proposed.

In this work, we only concern the OZI-suppressed processes for $J/\psi(\Upsilon) \to \pi\pi,\ \rho\pi$, and will explicitly demonstrate the isospin conserving and violating effects and the helicity violation effects in the formulation.

As indicated above, to evaluate the hadronic matrix elements, one has to deal with a convolution integrals over the distribution amplitudes of the concerned hadrons. Because J/ψ and Υ contain two heavy constituents, their bound-state effects can be simply expressed in terms of the wave functions at origin which can be easily obtained from the data of their leptonic decays. The distribution functions of the two produced light hadrons might cause uncertainties, even though as indicated above, for π and ρ mesons, they can be reduced to minimum. It is interesting to note that the OZI-suppressed process for $J/\psi(\Upsilon) \to \rho\pi$ is forbidden by the hadronic helicity conservation at the leading twist, if the quark mass is neglected at the loop calculations. However, it is not zero and the transition amplitudes of such processes must be proportional to m_q^2. In this work we only consider the contribution from the leading twist distribution amplitudes of the mesons and show that as $m_q \to 0$, the amplitudes would approaches zero, in other words, we confirm the statement that the hadronic helicity conservation forbids the process $J/\psi(\Upsilon) \to \rho\pi$ at the leading twist if quark mass is neglected.

Following the literature, we can trust the calculations to a relatively accurate level. A rough numerical estimate by changing the input parameters in the distribution functions and its forms given in literature, shows that the error can be of order of a few tens percents.

After this introduction, we give all the formulas where we carry out the four- and five-point Feynamn integrals to obtain the hard-scattering amplitude at the quark level. The isospin violation factor $m_u - m_d$ explicitly shows up for $J/\psi(\Upsilon) \to \pi\pi$, and for the helicity-violated $J/\psi(\Upsilon) \to \rho\pi$ the amplitude is also proportional to the light-quark masses, then one needs to convolute the hard kernel with the initial and final states, and the convolution integration results in the physical transition amplitude in Sec. II. In Sec. III, we carefully analyze the infrared behavior in $J/\psi(\Upsilon) \to \pi\pi,\ \rho\pi$ and convince ourselves that all Feynman diagrams are infrared-safe when the end-point behaviors of the wave functions are considered. In Sec. IV, we make a numerical evaluation of the decay rates of $J/\psi(\Upsilon) \to \pi^+\pi^-$ and $\rho^\pm\pi^\mp$ and some necessary input parameters are explicitly given. The last section is devoted to a simple discussion on the uncertainties in our calculation and possible contributions to these processes from other mechanisms and then draw our conclusion. Some tedious details are collected in the appendices.

II. THEORETICAL CALCULATION ON THE RATES OF $J/\psi(\Upsilon) \to \pi\pi, \rho\pi$

In this work, without invoking the so-called weak-binding approximation which was adopted in literature[2], we explicitly keep the masses of the heavy and light quarks at the concerned propagators, when derive the transition amplitudes. The amplitude is written as

$$\begin{aligned} \mathcal{A} &= H \otimes \Phi_{J/\psi(\Upsilon)} \otimes \Phi_{P_1} \otimes \Phi_{P_2} \\ H &= C \otimes \widetilde{H} \end{aligned} \tag{1}$$

where the factors C, $\widetilde{H}$, $\Phi_{J/\psi(\Upsilon),P_1,P_2}$ are the color factor, hard kernel and distribution amplitudes of mesons, respectively. And the labels P_1, P_2 denote the two produced mesons in the final state. Indeed, here the perturbative and non-perturbative parts are factorized and a convolution integral would associate them to result in the physical amplitude. The detailed expressions of the hard kernels are given in Appendix A.

Below, in Fig. 1, we present the relevant Feynman diagrams. In these figures, we only explicitly draw the typical diagrams. There exit their topologically deformed diagrams which are obtained by exchanging the connections of the gluon-lines in the loop to the light-quark-gluon vertices, namely the two gluon-lines cross with each other. We do not explicitly show them in Fig. 1 just for simplicity. But definitely, in our derivation the contributions from those diagrams are included.

The amplitude of $J/\psi(\Upsilon) \to P_1P_2$ can be divided into three categories which correspond to Fig. 1 (1a1) and (1a2), (1b1) and (1b2), (1c1) and (1c2) respectively. To make the text succinct we collect the detailed expressions of the

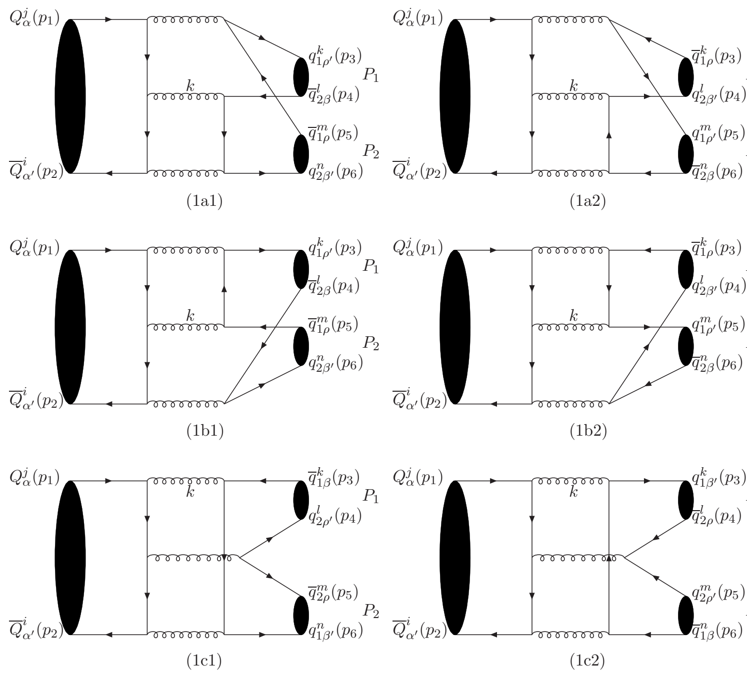

FIG. 1: The Feynman diagrams of $J/\psi(\Upsilon) \to P_1P_2$. Our calculation also includes such diagrams which are topologically deformed from that shown above by exchanging the connections of the gluon-lines in the loops to the gluon-light-quark vertices, where the two gluon-lines cross with each other.

amplitudes in Appendix B, where the diagrams with gluon-lines in the loops crossing each other are labelled as (2a1), (2a2), (2b1), (2b2), (2c1) and (2c2) respectively.

Generally, the transition amplitude for $J/\psi(\Upsilon) \to \pi^+\pi^-, \rho^+\pi^-, \rho^-\pi^+$ can be written as:

$$\begin{aligned}
\mathcal{A}^{J/\psi(\Upsilon)\to\pi^+\pi^-} &= \sum_i \mathcal{A}^i(P_1, P_2, m_q), \\
\mathcal{A}^{J/\psi(\Upsilon)\to\rho^+\pi^-(\rho^-\pi^+)} &= \sum_i (\mathcal{A}^{ia}(P_1 \to \rho, P_2 \to \pi, m_q) + \mathcal{A}^{ib}(P_1 \to \pi, P_2 \to \rho, m_q)),
\end{aligned} \tag{2}$$

where summing over i means including all diagrams listed above and their topologically deformed diagrams which were depicted above. For $\pi^+\pi^-$ final states one possible setting is that P_1, P_2 correspond to π^+, π^- respectively, and another possibility of interchanging π^+ and π^- is also included in the sum. m_q is either m_u or m_d, which respectively exist in different settings and their contributions should be summed in the final amplitude. For $\rho^+\pi^-$ or $\rho^-\pi^+$ final states interchanging P_1, P_2 would induce obvious differences and therefore we use two new labels "a" and "b" to distinguish the two different settings. Here we do not need to calculate the rate of $J/\psi(\Upsilon) \to \rho^0\pi^0$ because as discussed above, it is an isospin conserving process and the Clebsch-Gordan coefficients in I=0 state determines the ratio of $\Gamma(J/\psi(\Upsilon) \to \rho^0\pi^0)/\Gamma(J/\psi(\Upsilon) \to \rho\pi)$ and it should be close to 1/3, both the data and the analysis according to the topology of our diagrams shown in Fig. 1 confirm it, even though we only consider the contributions from the leading twist distribution amplitudes of the mesons.

In the quark picture, hadrons are made of valence quarks whose momenta-distributions are described by appropriate distribution functions. The leading-twist distribution amplitudes of $J/\psi(\Upsilon)$ is usually defined through the correlator[15]:

$$\begin{aligned}
\langle 0|\overline{c}^i_\alpha(y)c^j_\beta(x)|J/\psi(p)\rangle &= \frac{\delta_{ij}}{4N_c}\int_0^1 du e^{i\bar{u}p\cdot y + iup\cdot x} \times \\
&\left\{ f_{J/\psi} m_{J/\psi} \not{\varepsilon}_{J/\psi} \phi_{J/\psi\parallel}(u) + \frac{1}{2}\sigma^{\mu'\nu'} i f_{J/\psi}(\varepsilon_{J/\psi\mu'}p_{\nu'} - \varepsilon_{J/\psi\nu'}p_{\mu'})\phi_{J/\psi\perp}(u) \right\}_{\beta\alpha},
\end{aligned} \tag{3}$$

where $\varepsilon_{J/\psi}$ and $f_{J/\psi}$ are the polarization vector and decay constant of J/ψ respectively, and $\bar{u} \equiv 1-u$. $\phi_\parallel$ and $\phi_\perp$ are the leading-twist distribution functions corresponding to the longitudinally and transversely polarized mesons,

respectively, by the definition given in literature[15]. For the case of Υ one only needs to replace all the symbols of charm c into bottom b (of course as well as the concerned parameters). The leading-twist distribution amplitude of π is usually defined through the correlator[16]:

$$\langle\pi(p')|\overline{q}^i_{1\alpha}(y)q^j_{2\beta}(x)|0\rangle = i\frac{\delta_{ij}f_\pi}{4N_c}\int_0^1 du e^{iup'\cdot y+i\bar{u}p'\cdot x}\left\{\not{p}'\gamma_5\phi(u)\right\}_{\beta\alpha}, \tag{4}$$

where f_π is the decay constant of pion. And the leading-twist distribution amplitude of ρ is usually defined through the correlator[15]:

$$\begin{aligned}&\langle\rho(p')|\overline{q}^i_{1\alpha}(y)q^j_{2\beta}(x)|0\rangle = \frac{\delta_{ij}}{4N_c}\int_0^1 du e^{iup'\cdot y+i\bar{u}p'\cdot x}\times \\ &\left\{f_\rho m_\rho \not{\varepsilon}^*_\rho\phi_{\rho\|}(u)-\frac{1}{2}\sigma^{\mu'\nu'}if^T_\rho(\varepsilon^*_{\rho\mu'}p'_{\nu'}-\varepsilon^*_{\rho\nu'}p'_{\mu'})\phi_{\rho\perp}(u)\right\}_{\beta\alpha}\end{aligned} \tag{5}$$

where ε_ρ and f_ρ, f^T_ρ are the polarization vector and decay constant of ρ respectively.

III. ANALYSIS ON THE INFRARED BEHAVIORS IN $J/\psi(\Upsilon)\to P_1P_2$

It is definitely demanded that a reasonable theoretical prediction on any practical process must be infrared safe, namely the infrared divergence must be exactly cancels if it exists or properly dealt with at the end of the calculation which corresponds to real physical measurable quantities, such as the decay width and cross section. In this work, we will explicitly show that in the framework of perturbative QCD, the infrared behavior of each individual Feynman diagram shown in Fig. 1 is benign, even though at first glimpse it seems to be divergent.

There are several typical Feynman diagrams shown in Fig. 1. We take the amplitude of Fig. 1 (1a2) for $J/\psi\to P_1P_2$ as an example to analyze the infrared behavior.

Its contribution to the transition amplitude reads

$$\mathcal{A}^{1a2}(m_q,u,v) = H^{1a2}(m_q,u,v)\otimes\Phi_{P_1}(u)\otimes\Phi_{P_2}(v)., \tag{6}$$

and $H^{1a2}(m_q,u,v)$ is given in Appendix A. The concerned factor of the amplitude is

$$\frac{1}{k^2(k+p_4+p_6)^2[(k+p_4)^2-m_q^2][(k+p_1-p_3-p_5)^2-m_Q^2](p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]}. \tag{7}$$

Firstly, if there exists an infrared divergence in the loop integration, it must come from the kinematic region $k\to 0$ and the end-points of the distribution functions, therefore one only needs to analyze two cases: (1) $k\to 0$, (2) at end-points.

To show the infrared behavior of the amplitude after integrating out the loop function in the case (1), we may fix the external momenta of the quarks and antiquarks in final states by a special choice $p_3=p_4=\frac{1}{2}p_{P_1}, p_5=p_6=\frac{1}{2}p_{P_2}$ to avoid possible endpoint divergence. Looking at the expression (7), as $m_q\neq 0$, the dangerous part is proportional to $1/k^2$ which is finite after integrating over the loop momentum d^4k, so that in this case there is no infrared divergence coming from the loop integration.

Secondly, in case (2), when the momentum of one quark(antiquark) in each of the final mesons is close to its endpoint, for example $p_4,\ p_6\to 0$, while the other quark (antiquark) takes almost all the momentum of the meson. It is observed that the factor $\frac{1}{(p_3+p_5)^2}$ does not contribute a divergence. The dangerous term comes from the factor $k^2(k+p_4+p_6)^2$ at the denominator as $k\to 0$ and $p_4,\ p_6\to 0$, which seems to cause a logarithmic divergence. However as we convolute the amplitude with the distribution functions of the two mesons whose distribution functions linearly approach to $0(\phi(u),\phi(v)\to 0)$ at the end-points, because it turns to zero fasters than the logarithmically divergent factor, i.e. $\lim_{u\to 0}u\ln u\to 0$, the infrared behavior is safe.

Finally, when $p_3,\ p_5\to 0$, the loop integration does not produce any divergence by the same sake of the first case. It is noted, that the factor $\frac{1}{(p_3+p_5)^2}\phi(u)\phi(v)$ is finite, but there exists a subtlety. Namely in general the limit depends on the ways how x,y approach to 0 more or less. For $J/\psi(\Upsilon)\to\pi\pi$ there are two possible settings for each diagram, namely, an interchange $\pi^\pm\leftrightarrow\pi^\mp$ brings one setting to another, and their contributions have an opposite sign due to the SU(2) symmetry and cancels each other (obviously, for the finite term, their contributions cannot cancel each other due to the SU(2) breaking i.e. $m_u\neq m_d$). Thus the dependence on the order of limits disappears. By contraries, for $J/\psi(\Upsilon)\to\rho\pi$, there is no such a cancellation, so that even though infrared divergence does not exist, the final numerical results somehow depend on the order of taking limits of u and v approaching to zero. The strategy we adopt in this work is to set the integration order as we integrate over u and then v and it can be treated as a regularization scheme similar to that we generally adopt for treating the ultraviolet divergence.

IV. NUMERICAL RESULTS

The input parameters which we are going to use in the numerical computations are [5, 14, 15, 17, 18]: $f_{J/\psi} = 551$ MeV, $f_{\Upsilon} = 710$ MeV, $f_\pi = 131$ MeV, $f_\rho = 198$ MeV, $f_\rho^T = 160$ MeV, $m_{J/\psi} = 3096.87$ MeV, $m_\Upsilon = 9460.3$ MeV, $m_{\pi^\pm} = 139.57$ MeV, $m_{\rho^\pm} = 775.5$ MeV, $\alpha_s(m_c) = 0.32$, $\alpha_s(m_b) = 0.21$, $m_c = 1300$ MeV, $m_b = 4500$ MeV, and the meson distribution functions respectively. For the numerical evaluations, in Eqs.(4, 5), we adopt three different distribution functions for the light mesons in the literatures[16, 19, 20, 21] as $\phi_1(x) = 6x(1-x), \phi_2(x) = 30x^2(1-x)^2, \phi_3(x) = \frac{15}{2}(1-2x)^2[1-(1-2x)^2]$, and also let the current quark masses of the u and d types vary within a reasonable range.

Below in Tables I, II, III and IV, we present our numerical results on the decay rates of $J/\psi \to \pi^+\pi^-,\ \rho^+\pi^- + \rho^-\pi^+$ and $\Upsilon \to \pi^+\pi^-,\ \rho^+\pi^- + \rho^-\pi^+$ respectively.

TABLE I: Decay widths (Γ) of $J/\psi \to \pi^+\pi^-$ based on the three distribution functions, ϕ_1, ϕ_2 and ϕ_3, respectively

m_u(MeV)	m_d(MeV)	$\Gamma(\phi_1)$(MeV)	$\Gamma(\phi_2)$(MeV)	$\Gamma(\phi_3)$(MeV)	exp(MeV)
2	4	4.52×10^{-5}	2.98×10^{-5}	2.71×10^{-4}	
3	4	1.88×10^{-5}	9.35×10^{-6}	5.67×10^{-5}	
3	5	3.17×10^{-5}	2.36×10^{-5}	1.25×10^{-4}	$(1.37 \pm 0.21) \times 10^{-5}$
4	5	1.03×10^{-5}	8.12×10^{-6}	4.26×10^{-5}	
4.5	6	2.29×10^{-5}	1.43×10^{-5}	8.85×10^{-5}	

TABLE II: Decay widths (Γ) of $\Upsilon \to \pi^+\pi^-$ based on the three distribution functions, ϕ_1, ϕ_2 and ϕ_3, respectively

m_u(MeV)	m_d(MeV)	$\Gamma(\phi_1)$(MeV)	$\Gamma(\phi_2)$(MeV)	$\Gamma(\phi_3)$(MeV)	exp(MeV)
2	4	2.79×10^{-6}	1.24×10^{-6}	1.13×10^{-5}	
3	4	8.16×10^{-7}	5.28×10^{-7}	6.95×10^{-6}	
3	5	1.23×10^{-6}	9.43×10^{-7}	9.6×10^{-6}	$< 2.7 \times 10^{-5}$
4	5	7.39×10^{-7}	2.72×10^{-7}	5.11×10^{-6}	
4.5	6	9.78×10^{-7}	7.5×10^{-7}	8.93×10^{-6}	

TABLE III: Decay widths (Γ) of $J/\psi \to \pi^+\rho^- + \pi^-\rho^+$ based on the three distribution functions, ϕ_1, ϕ_2 and ϕ_3, respectively

m_u(MeV)	m_d(MeV)	$\Gamma(\phi_1)$(MeV)	$\Gamma(\phi_2)$(MeV)	$\Gamma(\phi_3)$(MeV)	exp(MeV)
2	2	1.04×10^{-4}	7.21×10^{-5}	5.11×10^{-4}	
3	3	2.36×10^{-4}	1.6×10^{-4}	1.17×10^{-3}	
4	4	4.12×10^{-4}	2.9×10^{-4}	2.08×10^{-3}	$(1.06 \pm 0.08) \times 10^{-3}$
5	5	6.69×10^{-4}	4.54×10^{-4}	3.38×10^{-3}	
6	6	9.75×10^{-4}	6.68×10^{-4}	4.88×10^{-3}	

As discussed above the OZI-suppressed process $J/\psi(\Upsilon) \to \pi^+\pi^-$ is isospin-violated whereas $J/\psi(\Upsilon) \to \pi^+\rho^- + \pi^-\rho^+$ violates the hadronic helicity conservation. It is noted that the theoretically evaluated values on the OZI-suppressed processes $J/\psi \to \pi^+\pi^-$ are slightly larger than the experimental data depending on the parameter choices such as m_u, m_d and types of the meson distribution functions, whereas that on $J/\psi \to \pi^+\rho^- + \pi^-\rho^+$ are one order smaller than the data. It may imply that some other mechanisms may also contribute to the decays and we will remark on the results in next section.

TABLE IV: Decay widths (Γ) of $\Upsilon \to \pi^+\rho^- + \pi^-\rho^+$ based on the three distribution functions, ϕ_1, ϕ_2 and ϕ_3, respectively

m_u(MeV)	m_d(MeV)	$\Gamma(\phi_1)$(MeV)	$\Gamma(\phi_2)$(MeV)	$\Gamma(\phi_3)$(MeV)	exp(MeV)
2	2	2.23×10^{-6}	1.56×10^{-6}	8.54×10^{-6}	
3	3	5.04×10^{-6}	3.67×10^{-6}	1.84×10^{-5}	
4	4	8.95×10^{-6}	6.25×10^{-6}	3.4×10^{-5}	$< 1.08 \times 10^{-5}$
5	5	1.42×10^{-5}	9.93×10^{-6}	5.44×10^{-5}	
6	6	1.84×10^{-5}	1.38×10^{-5}	7.61×10^{-5}	

V. DISCUSSION AND CONCLUSION

In this work, we calculate the contributions of the so-called OZI forbidden processes to the decays $J/\psi(\Upsilon) \to \pi\pi, \rho\pi$. As we discussed in the introduction, the process $J/\psi(\Upsilon) \to \pi\pi$ is an isospin violating reaction, whereas $J/\psi(\Upsilon) \to \rho\pi$ is an isospin conserving one, on other aspect, the former one conserves the hadronic helicity whereas the latter one violates it. Our numerical results on $J/\psi(\Upsilon) \to \pi^+\pi^-$ are reasonably consistent with the data at order of magnitude, but the evaluated branching ratio of $J/\psi(\Upsilon) \to \rho\pi$ is obviously smaller than data by one order.

As shown in Tables I through IV, one can notice that the results deviate from each other in a wider range as one adopts different wave functions which all are suggested in literatures, as well as the light-quark masses.

$J/\psi(\Upsilon) \to \pi\pi$ is an isospin violating process and at the leading twist the OZI-suppressed process which was supposed to be the main contribution to the mode of $J/\psi(\Upsilon) \to \rho\pi$ violates the hadronic helicity conservation. As well known, the source of isospin violation can be either from a photon emission (absorption) and/or quark mass difference, and for the helicity violating processes, the decay width is proportional to m_q^2, so that both of the processes are somehow sensitive to the light quark masses and much suppressed. Our formulas explicitly show that as $m_q \to 0$, the decay widths for both modes approach zero. This observation confirms the above statements.

In this work, we only include the contributions from the leading-twist distribution amplitude and our results confirm that due to violation of helicity conservation, the theoretical evaluated ratio is one order of magnitude smaller than the data. It is also noted from our qualitative analysis that the rate of isospin-violated process $J/\psi(\Upsilon) \to \pi\pi$ should be proportional to the square of mass difference $(m_u - m_d)^2$, whereas rate of the the hadronic helicity-violating process $J/\psi(\Upsilon) \to \rho\pi$ is proportional to $(m_u + m_d)^2$, i.e. it is natural to expect that $\Gamma(J/\psi(\Upsilon) \to \rho\pi)$ which is theoretically estimated in this framework, is a few times larger than $\Gamma(J/\psi(\Upsilon) \to \pi\pi)$, our numerical results shown in Tables I through IV confirm this statement, and if $m_u = m_d$, the estimated $\Gamma(J/\psi(\Upsilon) \to \pi\pi)$ is zero, whereas $\Gamma(J/\psi(\Upsilon) \to \rho\pi)$ is not. But this still does not explain the largeness of the branching ratio of $J/\psi(\Upsilon) \to \rho\pi$. It is indicated in Refs.[12, 13], the large branching ratio might be due to higher twist contributions. Therefore it seems that to correctly evaluate the branching ratio, in principle one needs to include the contributions from higher twist distribution amplitudes in the evaluation.

On other side, besides the contributions from higher twist distribution amplitudes, there may exist other mechanisms which may result in larger branching ratios for $J/\psi(\Upsilon) \to \rho\pi$. As suggested by Suzuki paper[22] and our earlier work[7], there can be a contribution from the hadronic loops and by fitting data (in the paper, the contributions from the OZI-forbidden processes were not theoretically calculated as we do in this work, but obtained by fitting data), we reached two conclusions that if only the two mechanisms contributing, the hadronic loop contribution would have the same order of magnitudes as that of the OZI-forbidden processes (definitely including higher twist contributions) and secondly the two contributions are destructive.

Of course, it may not be the end of the story that some authors also suggested a glueball contribution which should be added to that from the aforementioned mechanisms[6], and then the picture becomes more complicated, because we are unable to reliably estimate the glueball mass and phenomenological behaviors so far, unless we can borrow the lattice results. Therefore further developments on theory are necessary.

Uncertainties in our theoretical evaluations come from the input parameters, especially the light quark masses and the shapes of the distribution functions while only the leading-twist distribution amplitudes are accounted. One can note that the shapes of the wave functions would cause order of magnitude differences. So far, we still cannot really rule out any of them, but wait for more accurate data to determine.

As indicated in the text, we only consider the processes of $J/\psi(\Upsilon) \to \pi\pi,\ \rho\pi$ because as there no strange flavor gets involved, the wave functions of the produced mesons is simpler and more symmetric. For the processes involving such as $K(K^*),\ \eta,\ \eta'$, the calculations become more complicated and the results are not much reliable. Therefore we postpone our study on such processes in our later works.

So far, the experimental data are not accurate yet, especially for the measurements on Υ decays only upper limits

are set. However, we are inspired by the promises from the $\mathrm{CLEO_c}$ and BES III collaborations, as they will provide a much larger database on J/ψ decays, and more data would be accumulated in the B-meson factories, and then we will have concrete numbers about the branching ratios of $\Upsilon \to \pi\pi,\ \rho\pi$ instead of the upper limits set by the present experimental measurements. Moreover, the $\mathrm{LHC_b}$ and future ILC can much enrich our knowledge on hadron structure. Conclusion is definite that further work is necessary.

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Appendix A: The hard-scattering amplitudes $H^{i,\alpha'\beta'\gamma'\rho'\alpha\beta\gamma\rho}(m_q,u,v)$

The hard-scattering amplitude corresponding to Fig. 1 (1a1) is:

$$\begin{aligned}
&H^{1a1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^n_{2\beta'}[(-ig_sT^a_{nw}\gamma_\nu)\frac{i}{-\not k-\not p_4-m_q}(-ig_sT^b_{wl}\gamma_\mu)]_{\beta'\beta}q^l_{2\beta}\overline{q}^k_{1\rho'}[(-ig_sT^c_{km}\gamma_\lambda)]_{\rho'\rho}q^m_{1\rho}\frac{-i}{(p_3+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_4+p_6)^2}
\end{aligned} \tag{8}$$

The hard-scattering amplitude corresponding to Fig. 1 (1a2) is:

$$\begin{aligned}
&H^{1a2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^l_{2\beta'}[(-ig_sT^b_{lw}\gamma_\mu)\frac{i}{\not k+\not p_4-m_q}(-ig_sT^a_{wn}\gamma_\nu)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^c_{mk}\gamma_\lambda)]_{\rho'\rho}q^k_{1\rho}\frac{-i}{(p_3+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_4+p_6)^2}
\end{aligned} \tag{9}$$

The hard-scattering amplitude corresponding to Fig. 1 (1b1) is:

$$\begin{aligned}
&H^{1b1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^n_{2\beta'}[(-ig_sT^a_{nl}\gamma_\nu)]_{\beta'\beta}q^l_{2\beta}\overline{q}^k_{1\rho'}[(-ig_sT^c_{kw}\gamma_\lambda)\frac{i}{\not k-\not p_5-m_q}(-ig_sT^b_{wm}\gamma_\mu)]_{\rho'\rho}q^m_{1\rho}\frac{-i}{(p_4+p_6)^2}\frac{-i}{k^2}\frac{-i}{(k-p_3-p_5)^2}
\end{aligned} \tag{10}$$

The hard-scattering amplitude corresponding to Fig. 1 (1b2) is:

$$\begin{aligned}
&H^{1b2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^l_{2\beta'}[(-ig_sT^a_{ln}\gamma_\nu)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^b_{mw}\gamma_\mu)\frac{i}{-\not k+\not p_5-m_q}(-ig_sT^c_{wk}\gamma_\lambda)]_{\rho'\rho}q^k_{1\rho}\frac{-i}{(p_4+p_6')^2}\frac{-i}{k^2}\frac{-i}{(k-p_3-p_5)^2}
\end{aligned} \tag{11}$$

The hard-scattering amplitude corresponding to Fig. 1 (1c1) is:

$$\begin{aligned}
&H^{1c1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_4-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_1-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^n_{2\beta'}[(-ig_sT^a_{nw}\gamma_\nu)\frac{i}{-\not k-\not p_3-m_q}(-ig_sT^c_{wk}\gamma_\lambda)]_{\beta'\beta}q^k_{2\beta}\overline{q}^l_{1\rho'}[(-ig_sT^b_{lm}\gamma_\mu)]_{\rho'\rho}q^m_{1\rho} \\
&\frac{-i}{(p_4+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_1+p_2-p_4-p_5)^2}
\end{aligned} \tag{12}$$

The hard scattering amplitude corresponding to Fig. 1 (1c2) is:

$$H^{1c2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_4-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_1-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^k_{2\beta'}[(-ig_sT^c_{kw}\gamma_\lambda)\frac{i}{\not k+\not p_3-m_q}(-ig_sT^a_{wn}\gamma_\nu)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^b_{ml}\gamma_\mu)]_{\rho'\rho}q^l_{1\rho}$$
$$\frac{-i}{(p_4+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_1+p_2-p_4-p_5)^2} \tag{13}$$

The hard-scattering amplitude corresponding to the diagram which is topologically deformed from Fig. 1 (1a1) by exchanging the connection of the gluon-lines in the loop to the gluon-light-quark vertices, is

$$H^{2a1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^n_{2\beta'}[(-ig_sT^b_{nw}\gamma_\mu)\frac{i}{\not k+\not p_6-m_q}(-ig_sT^a_{wl}\gamma_\nu)]_{\beta'\beta}q^l_{2\beta}\overline{q}^k_{1\rho'}[(-ig_sT^c_{km}\gamma_\lambda)]_{\rho'\rho}q^m_{1\rho}\frac{-i}{(p_3+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_4+p_6)^2} \tag{14}$$

The hard-scattering amplitude corresponding to the topologically deformed diagram from Fig. 1 (1a2) is:

$$H^{2a2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not p_1-\not p_3-\not p_5-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^l_{2\beta'}[(-ig_sT^a_{lw}\gamma_\nu)\frac{i}{-\not k-\not p_6-m_q}(-ig_sT^b_{wn}\gamma_\mu)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^c_{mk}\gamma_\lambda)]_{\rho'\rho}q^k_{1\rho}\frac{-i}{(p_3+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_4+p_6)^2} \tag{15}$$

The hard-scattering amplitude corresponding to the deformed diagram from Fig. 1 (1b1) is:

$$H^{2b1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^n_{2\beta'}[(-ig_sT^a_{nl}\gamma_\nu)]_{\beta'\beta}q^l_{2\beta}\overline{q}^k_{1\rho'}[(-ig_sT^b_{kw}\gamma_\mu)\frac{i}{-\not k+\not p_3-m_q}(-ig_sT^c_{wm}\gamma_\lambda)]_{\rho'\rho}q^m_{1\rho}\frac{-i}{(p_4+p_6)^2}\frac{-i}{k^2}\frac{-i}{(k-p_3-p_5)^2} \tag{16}$$

The hard-scattering amplitude corresponding to the deformed diagram from Fig. 1 (1b2) is:

$$H^{2b2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_4+\not p_6-\not p_2-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^l_{2\beta'}[(-ig_sT^a_{ln}\gamma_\nu)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^c_{mw}\gamma_\lambda)\frac{i}{\not k-\not p_3-m_q}(-ig_sT^b_{wk}\gamma_\mu)]_{\rho'\rho}q^k_{1\rho}\frac{-i}{(p_4+p_6)^2}\frac{-i}{k^2}\frac{-i}{(k-p_3-p_5)^2} \tag{17}$$

The hard-scattering amplitude corresponding to the deformed diagram from Fig. 1 (1c1) is:

$$H^{2c1,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) =$$
$$\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_4-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_1-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha$$
$$\overline{q}^n_{2\beta'}[(-ig_sT^c_{nw}\gamma_\lambda)\frac{i}{\not k+\not p_6-m_q}(-ig_sT^a_{wk}\gamma_\nu)]_{\beta'\beta}q^k_{2\beta}\overline{q}^l_{1\rho'}[(-ig_sT^b_{lm}\gamma_\mu)]_{\rho'\rho}q^m_{1\rho}$$
$$\frac{-i}{(p_4+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_1+p_2-p_4-p_5)^2} \tag{18}$$

The hard-scattering amplitude corresponding to the deformed diagram from Fig. 1 (1c2) is:

$$
\begin{aligned}
&H^{2c2,\alpha\alpha'\beta\beta'\rho\rho'}(m_q,u,v) = \\
&\int \frac{d^4k}{(2\pi)^4}\overline{Q}^i_{\alpha'}[(-ig_sT^a_{is}\gamma^\nu)\frac{i}{\not k+\not p_1-\not p_4-\not p_5-m_Q}(-ig_sT^b_{sr}\gamma^\mu)\frac{i}{\not k+\not p_1-m_Q}(-ig_sT^c_{rj}\gamma^\lambda)]_{\alpha'\alpha}Q^j_\alpha \\
&\overline{q}^k_{2\beta'}[(-ig_sT^a_{kw}\gamma_\nu)\frac{i}{-\not k-\not p'_6-m_q}(-ig_sT^c_{wn}\gamma_\lambda)]_{\beta'\beta}q^n_{2\beta}\overline{q}^m_{1\rho'}[(-ig_sT^b_{ml}\gamma_\mu)]_{\rho'\rho}q^l_{1\rho} \\
&\frac{-i}{(p_4+p_5)^2}\frac{-i}{k^2}\frac{-i}{(k+p_1+p_2-p_4-p_5)^2}
\end{aligned} \tag{19}
$$

Appendix B: The amplitudes $\mathcal{A}^i(m_q,u,v)$

1. For $\mathbf{J/\psi\to PP}$

For amplitudes $\mathcal{A}^{1a1}$ and $\mathcal{A}^{1a2}$, we have

$$
\begin{aligned}
\mathcal{A}^{1a1} &= C^{1a1}\widetilde{H}^{1a1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{1a2} &= C^{1a2}\widetilde{H}^{1a2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2}
\end{aligned} \tag{20}
$$

with

$$
\begin{aligned}
C^{1a1} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
C^{1a2} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
\widetilde{H}^{1a1}(m_q,u,v) &= -\widetilde{H}^{1a2}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
&\{D_0(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p_{P2}m_Q^2+96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p_{P1}m_Q^2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p_{P1}\cdot p_{P2}m_Q^2+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p_{P2}p'^2_3-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p_{P1}p'^2_3 \\
&-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p'_3p'_3\cdot p_{P2}+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p'_1\cdot p_{P1}p'_3\cdot p_{P2} \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p'_3p'_3\cdot p_{P1}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_3\cdot p_{P2}p'_3\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^2_3p_{P1}\cdot p_{P2}] \\
&+D_\mu(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}m_Q^2+96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}m_Q^2 \\
&-32m_{J/\psi}\varepsilon^\mu_{J/\psi}p_{P1}\cdot p_{P2}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_1\cdot p'_3-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_1\cdot p'_3 \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_3p'_1\cdot p_{P2}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{P1}p'_1\cdot p_{P2}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_3p'_1\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{P2}p'_1\cdot p_{P1}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'^2_3-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'^2_3 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_1p'_3\cdot p_{P2}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_3p'_3\cdot p_{P2}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P1}p'_3\cdot p_{P2} \\
&+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{P1}p'_3\cdot p_{P2}+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P1}p'_3\cdot p_{P2}-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_1p'_3\cdot p_{P1} \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_3p'_3\cdot p_{P1}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P2}p'_3\cdot p_{P1}-32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P2}p'_3\cdot p_{P1} \\
&-64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_3\cdot p_{P2}p'_3\cdot p_{P1}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p'^\mu_1p_{P2}\cdot p_{P1}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_3p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p'_3p_{P2}\cdot p_{P1}+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^2_3p_{P2}\cdot p_{P1}] \\
&+D_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_3p^\nu_{P1}+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{P2}p^\nu_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_3\cdot p_{P2}-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_3\cdot p_{P1}-64m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P2}p'_3\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3g^{\mu\nu}p_{P2}\cdot p_{P1}+64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^\nu_3p_{P2}\cdot p_{P1}]\} \\
D_0(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{1}{k^2[(k+p_4)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]} \\
D_\mu(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu}{k^2[(k+p_4)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]} \\
D_{\mu\nu}(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu k_\nu}{k^2[(k+p_4)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]}
\end{aligned} \tag{21}
$$

where $p_1' = p_4, p_3' = p_1 - p_3 - p_5$.

For amplitudes $\mathcal{A}^{1b1}$ and $\mathcal{A}^{1b2}$, we have

$$\begin{aligned}
\mathcal{A}^{1b1} &= C^{1b1}\widetilde{H}^{1b1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{1b2} &= C^{1b2}\widetilde{H}^{1b2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2}
\end{aligned} \tag{22}$$

with

$$\begin{aligned}
C^{1b1} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
C^{1b2} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{1b1}(m_q,u,v) &= -\widetilde{H}^{1b2}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
&\{D_0(m_q,u,v)[96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_{P1}m_Q^2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P2}\cdot p_{P1}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_{P2}p_3'^2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_{P1}p_3'^2 \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_3'p_3'\cdot p_{P2} - 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_3'p_3'\cdot p_{P1} \\
&+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_1'\cdot p_{P2}p_3'\cdot p_{P1} - 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'\cdot p_{P2}p_3'\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'^2p_{P2}\cdot p_{P1}] \\
&+D_\mu(m_q,u,v)[96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu m_Q^2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_{P1}\cdot p_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu p_1'\cdot p_3' - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu p_1'\cdot p_3' \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_3'^\mu p_1'\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P1}^\mu p_1'\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_3'^\mu p_1'\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P2}^\mu p_1'\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu p_3'^2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu p_3'^2 \\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'^\mu p_3'\cdot p_{P2} - 128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_3'^\mu p_3'\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P1}^\mu p_3'\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_{P1}p_3'\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'^\mu p_3'\cdot p_{P1} - 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_3'^\mu p_3'\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P2}^\mu p_3'\cdot p_{P1} + 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P2}^\mu p_3'\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_{P2}p_3'\cdot p_{P1} \\
&-64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'\cdot p_{P2}p_3'\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_1'^\mu p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'^\mu p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_3'p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'^2p_{P2}\cdot p_{P1}] \\
&+D_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_3'^\mu p_{P2}^\nu + 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P2}^\mu p_{P1}^\nu \\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p_3'\cdot p_{P2} - 64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_{P1}^\nu p_3'\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p_3'\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'g^{\mu\nu}p_{P2}\cdot p_{P1} + 64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'^\nu p_{P1}\cdot p_{P2}]\} \\
D_0(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{1}{k^2[(k-p_5)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]} \\
D_\mu(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu}{k^2[(k-p_5)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]} \\
D_{\mu\nu}(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu k_\nu}{k^2[(k-p_5)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]}
\end{aligned} \tag{23}$$

where $p_1' = -p_5, p_3' = p_4 + p_6 - p_2$.

For amplitudes $\mathcal{A}^{1c1}$ and $\mathcal{A}^{1c2}$, we have

$$\begin{aligned}
\mathcal{A}^{1c1} &= C^{1c1}\widetilde{H}^{1c1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{1c2} &= C^{1c2}\widetilde{H}^{1c2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2}
\end{aligned} \tag{24}$$

with

$$
\begin{aligned}
&C^{1c1} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
&C^{1c2} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)\\
&\widetilde{H}^{1c1}(m_q,u,v) = -\widetilde{H}^{1c2}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}\\
&\{E_0(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_4\cdot p_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_4\cdot p_{P1}m_Q^2\\
&+96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p_{P2}\cdot p_{P1}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p_{P1}p'_2\cdot p'_4\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_2\cdot p_{P2}p'_1\cdot p'_4 - 96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p_{P1}p'_2\cdot p_{P2}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p'_4p'_2\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p_{P2}p'_2\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p'_2p'_4\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p_{P1}p'_4\cdot p_{P2}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p_{P1}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p'_2p'_4\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p_{P2}p'_4\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p_{P2}p'_4\cdot p_{P1}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p'_2p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p'_4p_{P2}\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p'_4p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p_{P2}p'_2\cdot p'_4]\\
&+E_\mu(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}m_Q^2\\
&+96m_{J/\psi}\varepsilon^\mu_{J/\psi}p_{P1}\cdot p_{P2}m_Q^2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_1\cdot p'_2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_1\cdot p'_2\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_1\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_1\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_2p'_1\cdot p_{P2}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p'_1\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P1}p'_1\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P1}p'_1\cdot p_{P2}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_2p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P2}p'_1\cdot p_{P1}\\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_2\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_2\cdot p'_4\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_1p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P1}p'_2\cdot p_{P2}\\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P1}p'_2\cdot p_{P2} - 96m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P1}p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_1p'_2\cdot p_{P1}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P2}p'_2\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p'_2\cdot p_{P1}\\
&-32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P2}p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_1p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_2p'_4\cdot p_{P2}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P1}p'_4\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P1}p'_4\cdot p_{P2} - 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P1}p'_4\cdot p_{P2}\\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p_{P1}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_1p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_2p'_4\cdot p_{P1}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P2}p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P2}p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P2}p'_4\cdot p_{P1}\\
&-32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p_{P2}p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'^\mu_1p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'^\mu_1p_{P2}\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_2p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'^\mu_2p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_4p_{P2}\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'^\mu_4p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p'_1p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_4\cdot p'_1p_{P2}\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_4\cdot p'_2p_{P2}\cdot p_{P1}]\\
&+E_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p^\nu_{P2} - 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p^\nu_{P1}\\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p^\nu_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_1\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_1\cdot p_{P1}\\
&-128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P2}p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_2\cdot p_{P2} - 128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P1}p'_2\cdot p_{P2}\\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_4\cdot p_{P1}\\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1g^{\mu\nu}p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2g^{\mu\nu}p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4g^{\mu\nu}p_{P2}\cdot p_{P1}\\
&-64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^\nu_4p_{P2}\cdot p_{P1}]\\
&+E_{\mu\nu\theta}(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p^\theta_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p^\theta_{P1}\\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}g^{\nu\theta}p_{P1}\cdot p_{P2} - 128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P1}p^\theta_{P2}]\}
\end{aligned}
\tag{25}
$$

$$\begin{aligned}
&E_0(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{1}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_3)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_\mu(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_3)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_{\mu\nu}(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu k_\nu}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_3)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_{\mu\nu\theta}(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu k_\nu k_\theta}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_3)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2}
\end{aligned} \tag{26}$$

where $p_1' = p_1, p_2' = p_1 - p_4 - p_5, p_4' = p_3$.

For amplitudes $\mathcal{A}^{2a1}$ and $\mathcal{A}^{2a2}$, we have

$$\begin{aligned}
\mathcal{A}^{2a1} &= C^{2a1}\widetilde{H}^{2a1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{2a2} &= C^{2a2}\widetilde{H}^{2a2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2}
\end{aligned} \tag{27}$$

with

$$
\begin{aligned}
C^{2a1} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
C^{2a2} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{2a1}(m_q,u,v) &= -\widetilde{H}^{2a2}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
&\quad \{D_0(m_q,u,v)[96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'_1\cdot p_{_{P2}}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p_{_{P1}}\cdot p_{_{P2}}m_Q^2 \\
&\quad -32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'_1\cdot p_{_{P1}}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'_1\cdot p_{_{P2}}p'^2_3+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'_1\cdot p_{_{P1}}p'^2_3 \\
&\quad -128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'_1\cdot p'_3p'_3\cdot p_{_{P2}}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'_1\cdot p'_3p'_3\cdot p_{_{P1}} \\
&\quad +64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p'_1\cdot p_{_{P2}}p'_3\cdot p_{_{P1}}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_3\cdot p_{_{P2}}p'_3\cdot p_{_{P1}} \\
&\quad +32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^2_3p_{_{P2}}\cdot p_{_{P1}}] \\
&\quad D_\mu(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p^\mu_{_{P1}}m_Q^2+96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p^\mu_{_{P2}}m_Q^2 \\
&\quad -32m_{J/\psi}\varepsilon^\mu_{J/\psi}p_{_{P1}}\cdot p_{_{P2}}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p^\mu_{_{P2}}p'_1\cdot p'_3-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p^\mu_{_{P1}}p'_1\cdot p'_3 \\
&\quad -32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'^\mu_3p'_1\cdot p_{_{P2}}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{_{P1}}p'_1\cdot p_{_{P2}}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'^\mu_3p'_1\cdot p_{_{P1}} \\
&\quad +32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{_{P2}}p'_1\cdot p_{_{P1}}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p^\mu_{_{P2}}p'^2_3+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p^\mu_{_{P1}}p'^2_3 \\
&\quad -96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'^\mu_1p'_3\cdot p_{_{P2}}-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'^\mu_3p'_3\cdot p_{_{P2}}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{_{P1}}p'_3\cdot p_{_{P2}} \\
&\quad -32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{_{P1}}p'_3\cdot p_{_{P2}}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'^\mu_1p'_3\cdot p_{_{P1}}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}p'^\mu_3p'_3\cdot p_{_{P1}} \\
&\quad -32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{_{P2}}p'_3\cdot p_{_{P1}}+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{_{P2}}p'_3\cdot p_{_{P1}}+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{_{P2}}p'_3\cdot p_{_{P1}} \\
&\quad -64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_3\cdot p_{_{P2}}p'_3\cdot p_{_{P1}}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p'^\mu_1p_{_{P2}}\cdot p_{_{P1}}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_3p_{_{P2}}\cdot p_{_{P1}} \\
&\quad +32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p'_3p_{_{P2}}\cdot p_{_{P1}}+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^2_3p_{_{P2}}\cdot p_{_{P1}}] \\
&\quad +D_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}p'^\mu_3p^\nu_{_{P2}}+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3p^\mu_{_{P1}}p^\nu_{_{P2}} \\
&\quad -96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P1}}g^{\mu\nu}p'_3\cdot p_{_{P2}}-64m_{J/\psi}\varepsilon^\mu_{J/\psi}\cdot p^\nu_{_{P1}}p'_3\cdot p_{_{P2}}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{_{P2}}g^{\mu\nu}p'_3\cdot p_{_{P1}} \\
&\quad -32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_3g^{\mu\nu}p_{_{P2}}\cdot p_{_{P1}}+64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^\nu_3p_{_{P2}}\cdot p_{_{P1}}]\} \\
D_0(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{1}{k^2[(k+p_6)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]} \\
D_\mu(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu}{k^2[(k+p_6)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]} \\
D_{\mu\nu}(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu k_\nu}{k^2[(k+p_6)^2-m_q^2](k+p_4+p_6)^2[(k+p_1-p_3-p_5)^2-m_Q^2]} \qquad (28)
\end{aligned}
$$

where $p'_1=p_6, p'_3=p_1-p_3-p_5$.

For amplitudes $\mathcal{A}^{2b1}$ and $\mathcal{A}^{2b2}$, we have

$$
\begin{aligned}
\mathcal{A}^{2b1} &= C^{2b1}\widetilde{H}^{2b1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{2b2} &= C^{2b2}\widetilde{H}^{2b2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \qquad (29)
\end{aligned}
$$

with

$$
\begin{aligned}
C^{2b1} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
C^{2b2} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
\widetilde{H}^{2b1}(m_q,u,v) &= -\widetilde{H}^{2b2}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
&\{D_0(m_q,u,v)[96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_{P1}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_{P2}m_Q^2 \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P2}\cdot p_{P1}m_Q^2+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_{P2}p_3'^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_{P1}p_3'^2 \\
&-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'\cdot p_3'p_3'\cdot p_{P2}+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_1'\cdot p_{P2}p_3'\cdot p_{P2} \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'\cdot p_3'p_3'\cdot p_{P1}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'\cdot p_{P2}p_3'\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'^2p_{P2}\cdot p_{P1}] \\
&+D_\mu(m_q,u,v)[96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu m_Q^2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_{P1}\cdot p_{P2}m_Q^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu p_1'\cdot p_3'-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu p_1'\cdot p_3' \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_3'^\mu p_1'\cdot p_{P2}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P1}^\mu p_1'\cdot p_{P2}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_3'^\mu p_1'\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P2}^\mu p_1'\cdot p_{P1}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_{P2}^\mu p_3'^2-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu p_3'^2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_1'^\mu p_3'\cdot p_{P2}-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p_3'^\mu p_3'\cdot p_{P2}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P1}^\mu p_3'\cdot p_{P2} \\
&+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P1}^\mu p_3'\cdot p_{P2}+32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_{P1}p_3'\cdot p_{P2}-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_1'^\mu p_3'\cdot p_{P1} \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_3'^\mu p_3'\cdot p_{P1}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_{P2}^\mu p_3'\cdot p_{P1}-32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_{P2}p_3'\cdot p_{P1} \\
&-64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'\cdot p_{P2}p_3'\cdot p_{P1}-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_1'^\mu p_{P2}\cdot p_{P1}+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_1'p_3'^\mu p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_1'\cdot p_3'p_{P2}\cdot p_{P1}+32m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'^2p_{P2}\cdot p_{P1}] \\
&+D_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p_{P1}^\mu p_3'^\nu+64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'p_{P2}^\mu p_{P1}^\nu \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p_3'\cdot p_{P2}-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p_3'\cdot p_{P1}-64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_{P2}^\nu p_3'\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_3'g^{\mu\nu}p_{P2}\cdot p_{P1}+64m_{J/\psi}\varepsilon_{J/\psi}^\mu p_3'^\nu p_{P1}\cdot p_{P2}]\} \\
D_0(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{1}{k^2[(k-p_3)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]} \\
D_\mu(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu}{k^2[(k-p_3)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]} \\
D_{\mu\nu}(m_q,u,v) &= \frac{1}{i\pi^2}\int d^4k\frac{k_\mu k_\nu}{k^2[(k-p_3)^2-m_q^2](k-p_3-p_5)^2[(k+p_4+p_6-p_2)^2-m_Q^2]} \qquad (30)
\end{aligned}
$$

where $p_1'=-p_3, p_3'=p_4+p_6-p_2$.

For amplitudes $\mathcal{A}^{2c1}$ and $\mathcal{A}^{2c2}$, we have

$$
\begin{aligned}
\mathcal{A}^{2c1} &= C^{2c1}\widetilde{H}^{2c1}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \\
\mathcal{A}^{2c2} &= C^{2c2}\widetilde{H}^{2c2}(m_q,u,v)\Phi_{J/\psi}\Phi_{P1}\Phi_{P2} \qquad (31)
\end{aligned}
$$

with

$$C^{2c1} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)$$
$$C^{2c2} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)$$
$$\begin{aligned}
&\widetilde{H}^{2c1}(m_q,u,v) = -\widetilde{H}^{2c2}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2} \\
&\{E_0(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_4\cdot p_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_4\cdot p_{P1}m_Q^2 \\
&+96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p_{P2}\cdot p_{P1}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p_{P2}p'_2\cdot p'_4 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p_{P1}p'_2\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p'_4p'_2\cdot p_{P2} \\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p_{P1}p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p'_4p'_2\cdot p_{P1} \\
&-96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p_{P2}p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'_1\cdot p'_2p'_4\cdot p_{P2} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p_{P1}p'_4\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p_{P1}p'_4\cdot p_{P2} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'_1\cdot p'_2p'_4\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p_{P2}p'_4\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p_{P2}p'_4\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'_1\cdot p'_2p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'_1\cdot p'_4p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'_2\cdot p'_4p_{P2}\cdot p_{P1}] \\
&+E_\mu(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}m_Q^2 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}m_Q^2 \\
&+96m_{J/\psi}\varepsilon^\mu_{J/\psi}p_{P1}\cdot p_{P2}m_Q^2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_1\cdot p'_2 + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_1\cdot p'_2 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_1\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_1\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_2p'_1\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p'_1\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P1}p'_1\cdot p_{P2} - 96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P1}p'_1\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_2p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p'_1\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P2}p'_1\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p'_1\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p^\mu_{P2}p'_2\cdot p'_4 - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p^\mu_{P1}p'_2\cdot p'_4 \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_1p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p'_2\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P1}p'_2\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P1}p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P1}p'_2\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_1p'_2\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p'_2\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P2}p'_2\cdot p_{P1} - 96m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p'_2\cdot p_{P1} \\
&-96m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P2}p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_1p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_2p'_4\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P1}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P1}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P1}p'_4\cdot p_{P2} \\
&-32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p_{P1}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_1p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_2p'_4\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p^\mu_{P2}p'_4\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p^\mu_{P2}p'_4\cdot p_{P1} - 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_1\cdot p_{P2}p'_4\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p_{P2}p'_4\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'^\mu_1p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'^\mu_1p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_2p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p'^\mu_2p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1p'^\mu_4p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2p'^\mu_4p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_2\cdot p'_1p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_4\cdot p'_1p_{P2}\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}p'_4\cdot p'_2p_{P2}\cdot p_{P1}] \\
&+E_{\mu\nu}(m_q,u,v)[-64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}p'^\mu_4p^\nu_{P2} - 64m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}p'^\mu_4p^\nu_{P1} \\
&-128m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4p^\mu_{P2}p^\nu_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_1\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_1\cdot p_{P1} \\
&-128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P1}p'_1\cdot p_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_2\cdot p_{P2} - 128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P2}p'_2\cdot p_{P1} \\
&-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_2\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p'_4\cdot p_{P2} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p'_4\cdot p_{P1} \\
&+32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_1g^{\mu\nu}p_{P2}\cdot p_{P1} + 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_2g^{\mu\nu}p_{P2}\cdot p_{P1} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p'_4g^{\mu\nu}p_{P2}\cdot p_{P1} \\
&-64m_{J/\psi}\varepsilon^\mu_{J/\psi}p'^\nu_4p_{P2}\cdot p_{P1}] \\
&+E_{\mu\nu\theta}(m_q,u,v)[-32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P1}g^{\mu\nu}p^\theta_{P2} - 32m_{J/\psi}\varepsilon_{J/\psi}\cdot p_{P2}g^{\mu\nu}p^\theta_{P1} \\
&+32m_{J/\psi}\varepsilon^\mu_{J/\psi}g^{\nu\theta}p_{P1}\cdot p_{P2} - 128m_{J/\psi}\varepsilon^\mu_{J/\psi}p^\nu_{P1}p^\theta_{P2}]\}
\end{aligned} \tag{32}$$

$$
\begin{aligned}
&E_0(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{1}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_6)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_\mu(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_6)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_{\mu\nu}(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu k_\nu}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_6)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2} \\
&E_{\mu\nu\theta}(m_q,u,v) = \\
&\frac{1}{i\pi^2}\int d^4k \frac{k_\mu k_\nu k_\theta}{k^2[(k+p_1)^2-m_Q^2][(k+p_1-p_4-p_5)^2-m_Q^2][(k+p_6)^2-m_q^2](k+p_1+p_2-p_4-p_5)^2}
\end{aligned} \tag{33}
$$

where $p_1'=p_1, p_2'=p_1-p_4-p_5, p_4'=p_6$.

2. The case of $\mathbf{J/\psi \to VP}$

For amplitude $\mathcal{A}^{1a1a}$, we have

$$
\mathcal{A}^{1a1a} = C^{1a1a}\widetilde{H}^{1a1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{34}
$$

with

$$
\begin{aligned}
C^{1a1a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{1a1a}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
& m_Q m_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p_3'\cdot p_{J/\psi} - 64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p_3'\cdot p_P \\
& +64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p_3'\cdot p_V + 64\varepsilon^\nu_{J/\psi}\varepsilon^{*\alpha}_V p_3'^\mu p^\beta_P p_{J/\psi}\cdot p_V - 64\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_3'^\nu p_V\cdot p_P] \\
& +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p^\theta_{J/\psi} + 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p^\theta_P + 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p^\theta_V) \\
& +D^\nu(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_V p_{J/\psi}\cdot p_P - 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_P p_{J/\psi}\cdot p_V \\
& -32\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{35}
$$

where $p_1'=p_4, p_3'=p_1-p_3-p_5$.

For amplitude $\mathcal{A}^{1a1b}$, we have

$$
\mathcal{A}^{1a1b} = C^{1a1b}\widetilde{H}^{1a1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{36}
$$

with

$$
\begin{aligned}
C^{1a1b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{1a1b}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
& m_Q m_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p_3'\cdot p_{J/\psi} - 64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p_3'\cdot p_P \\
& +64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p_3'\cdot p_V - 64\varepsilon^\nu_{J/\psi}\varepsilon^{*\alpha}_V p_3'^\mu p^\beta_V p_{J/\psi}\cdot p_P + 64\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_3'^\nu p_V\cdot p_P] \\
& +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(96\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p^\theta_{J/\psi} - 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p^\theta_P - 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p^\theta_V) \\
& +D^\nu(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_V p_{J/\psi}\cdot p_P - 32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_P p_{J/\psi}\cdot p_V \\
& +32\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{37}
$$

where $p_1'=p_4, p_3'=p_1-p_3-p_5$.

For amplitude $\mathcal{A}^{1a2a}$, we have

$$\mathcal{A}^{1a2a} = C^{1a2a}\widetilde{H}^{1a2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{38}$$

with

$$\begin{aligned}
C^{1a2a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{1a2a}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]}\\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p'_3\cdot p_{J/\psi}+64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p'_3\cdot p_P\\
&\quad +64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p'_3\cdot p_V+64\varepsilon^\nu_{J/\psi}\varepsilon^{*\alpha}_V p'^\mu_3 p^\beta_P p_{J/\psi}\cdot p_V-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p'^\nu_3 p_V\cdot p_P]\\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p^\theta_{J/\psi}-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p^\theta_P+32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p^\theta_V)\\
&\quad +D^\nu(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_V p_{J/\psi}\cdot p_P-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_P p_{J/\psi}\cdot p_V\\
&\quad -32\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{39}$$

where $p'_1=p_4, p'_3=p_1-p_3-p_5$.

For amplitude $\mathcal{A}^{1a2b}$, we have

$$\mathcal{A}^{1a2b} = C^{1a2b}\widetilde{H}^{1a2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{40}$$

with

$$\begin{aligned}
C^{1a2b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{1a2b}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]}\\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p'_3\cdot p_{J/\psi}-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p'_3\cdot p_P\\
&\quad -64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p'_3\cdot p_V-64\varepsilon^\nu_{J/\psi}\varepsilon^{*\alpha}_V p'^\mu_3 p^\beta_V p_{J/\psi}\cdot p_P+64\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p'^\nu_3 p_V\cdot p_P]\\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p^\theta_{J/\psi}-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p^\theta_P+32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p^\theta_V)\\
&\quad +D^\nu(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_V p_{J/\psi}\cdot p_P+32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_P p_{J/\psi}\cdot p_V\\
&\quad +32\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{41}$$

where $p'_1=p_4, p'_3=p_1-p_3-p_5$.

For amplitude $\mathcal{A}^{1b1a}$, we have

$$\mathcal{A}^{1b1a} = C^{1b1a}\widetilde{H}^{1b1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{42}$$

with

$$\begin{aligned}
C^{1b1a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{1b1a}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]}\\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p'_3\cdot p_P+64\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p'_3\cdot p_V\\
&\quad +64\varepsilon^\nu_{J/\psi}\varepsilon^{*\alpha}_V p'^\mu_3 p^\beta_P p_{J/\psi}\cdot p_V-64\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p'^\nu_3 p^\alpha_{J/\psi}p_V\cdot p_P]\\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^\mu_{J/\psi}\varepsilon^{*\nu}_V p^\alpha_P p^\beta_V p^\theta_{J/\psi}-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_V p^\theta_P+32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\nu_{J/\psi}p^\beta_P p^\theta_V)\\
&\quad +D^\nu(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_V p_{J/\psi}\cdot p_P-32\varepsilon^\mu_{J/\psi}\varepsilon^{*\alpha}_V p^\beta_P p_{J/\psi}\cdot p_V\\
&\quad -32\varepsilon^\mu_{J/\psi}\varepsilon^{*\beta}_V p^\alpha_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{43}$$

where $p'_1=-p_5, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{1b1b}$, we have

$$\mathcal{A}^{1b1b} = C^{1b1b}\widetilde{H}^{1b1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{44}$$

with

$$\begin{aligned}
C^{1b1b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
\widetilde{H}^{1b1b}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V \\
& -64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{V}p_{J/\psi}\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\nu}_3p^{\alpha}_{J/\psi}p_V\cdot p_P] \\
& +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
& +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V \\
& +32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{45}$$

where $p'_1=-p_5, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{1b2a}$, we have

$$\mathcal{A}^{1b2a} = C^{1b2a}\widetilde{H}^{1b2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{46}$$

with

$$\begin{aligned}
C^{1b2a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{1b2a}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V \\
& +64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{P}p_{J/\psi}\cdot p_V-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\nu}_3p^{\alpha}_{J/\psi}p_V\cdot p_P] \\
& +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
& +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V \\
& -32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{47}$$

where $p'_1=-p_5, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{1b2b}$, we have

$$\mathcal{A}^{1b2b} = C^{1b2b}\widetilde{H}^{1b2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{48}$$

with

$$\begin{aligned}
C^{1b2b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{1b2b}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V \\
& -64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{V}p_{J/\psi}\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\nu}_3p^{\alpha}_{J/\psi}p_V\cdot p_P] \\
& +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
& +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V \\
& +32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{49}$$

where $p'_1=-p_5, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{1c1a}$, we have

$$\mathcal{A}^{1c1a} = C^{1c1a}\widetilde{H}^{1c1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{50}$$

with

$$
\begin{aligned}
&C^{1c1a} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
&\widetilde{H}^{1c1a}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2} \\
&m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_{V} \\
&-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_{V} \\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_{P} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_{V} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_{P}\cdot p_{V} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}] \\
&+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{51}
$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_3$.

For amplitude $\mathcal{A}^{1c1b}$, we have

$$\mathcal{A}^{1c1b} = C^{1c1b}\widetilde{H}^{1c1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{52}$$

with

$$
\begin{aligned}
&C^{1c1b} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c) \\
&\widetilde{H}^{1c1b}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2} \\
&m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_{P} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_{V} \\
&-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_{P} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_{V} \\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_{P} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_{V} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_{P}\cdot p_{V} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}] \\
&+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{53}
$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_3$.

For amplitude $\mathcal{A}^{1c2a}$, we have

$$\mathcal{A}^{1c2a} = C^{1c2a}\widetilde{H}^{1c2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{54}$$

with

$$
\begin{aligned}
&C^{1c2a} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
&\widetilde{H}^{1c2a}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2} \\
&m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_{V} \\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_{V} \\
&-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_{P} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_{P} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_{V} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_{P}\cdot p_{V} - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}] \\
&+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{55}
$$

where $p_1' = p_1, p_2' = p_1 - p_4 - p_5, p_4' = p_3$.

For amplitude $\mathcal{A}^{1c2b}$, we have

$$\mathcal{A}^{1c2b} \;=\; C^{1c2b}\widetilde{H}^{1c2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{56}$$

with

$$\begin{aligned}
&C^{1c2b} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)\\
&\widetilde{H}^{1c2b}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}\\
&m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p_2'\cdot p_{J/\psi}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_2'\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_2'\cdot p_V\\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p_1'\cdot p_{J/\psi}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_1'\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_1'\cdot p_V\\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_2'^{\mu}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_1'^{\mu}p^{\beta}_{V}p_{J/\psi}\cdot p_P+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_2'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_1'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_V+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p_2'^{\mu}p^{\alpha}_{J/\psi}p_P\cdot p_V+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p_1'^{\mu}p^{\alpha}_{J/\psi}p_P\cdot p_V]\\
&+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+E_1^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{57}$$

where $p_1' = p_1, p_2' = p_1 - p_4 - p_5, p_4' = p_3$.

For amplitude $\mathcal{A}^{2a1a}$, we have

$$\mathcal{A}^{2a1a} \;=\; C^{2a1a}\widetilde{H}^{2a1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{58}$$

with

$$\begin{aligned}
C^{2a1a} \;&=\; \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{2a1a}(m_q,u,v) \;&=\; \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]}\\
&m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_3'\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_3'\cdot p_V\\
&+64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_3'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_V-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p_3'^{\nu}p^{\alpha}_{J/\psi}p_P\cdot p_V]\\
&+[D_{\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{59}$$

where $p_1' = p_6, p_3' = p_1 - p_3 - p_5$.

For amplitude $\mathcal{A}^{2a1b}$, we have

$$\mathcal{A}^{2a1b} \;=\; C^{2a1b}\widetilde{H}^{2a1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{60}$$

with

$$\begin{aligned}
C^{2a1b} \;&=\; \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{2a1b}(m_q,u,v) \;&=\; \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]}\\
&m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_3'\cdot p_P+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_3'\cdot p_V\\
&-64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_3'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_V+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p_3'^{\nu}p^{\alpha}_{J/\psi}p_P\cdot p_V]\\
&+[D_{\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned} \tag{61}$$

where $p_1' = p_6, p_3' = p_1 - p_3 - p_5$.

For amplitude $\mathcal{A}^{2a2a}$, we have

$$\mathcal{A}^{2a2a} = C^{2a2a}\widetilde{H}^{2a2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{62}$$

with

$$\begin{aligned}
C^{2a2a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{2a2a}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_3'\cdot p_{P}+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_3'\cdot p_{V} \\
&\quad +64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_3'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_{V}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p_3'^{\nu}p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}] \\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&\quad +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&\quad -32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{63}$$

where $p_1' = p_6, p_3' = p_1 - p_3 - p_5$.

For amplitude $\mathcal{A}^{2a2b}$, we have

$$\mathcal{A}^{2a2b} = C^{2a2b}\widetilde{H}^{2a2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{64}$$

with

$$\begin{aligned}
C^{2a2b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{2a2b}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_3+p_5)^2[(p_1-p_3-p_5)^2-m_Q^2]} \\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_3'\cdot p_{P}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_3'\cdot p_{V} \\
&\quad -64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_3'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_{V}+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p_3'^{\nu}p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}] \\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&\quad +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&\quad +32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{65}$$

where $p_1' = p_6, p_3' = p_1 - p_3 - p_5$.

For amplitude $\mathcal{A}^{2b1a}$, we have

$$\mathcal{A}^{2b1a} = C^{2b1a}\widetilde{H}^{2b1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{66}$$

with

$$\begin{aligned}
C^{2b1a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c) \\
\widetilde{H}^{2b1a}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]} \\
&\quad m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p_3'\cdot p_{J/\psi}+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p_3'\cdot p_{P} \\
&\quad +64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p_3'\cdot p_{V}+64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p_3'^{\mu}p^{\beta}_{P}p_{J/\psi}\cdot p_{V}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_3'^{\nu}p_{V}\cdot p_{P}] \\
&\quad +[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V}) \\
&\quad +D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} \\
&\quad -32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{67}$$

where $p_1' = -p_3, p_3' = p_4 + p_6 - p_2$.

For amplitude $\mathcal{A}^{2b1b}$, we have

$$\mathcal{A}^{2b1b} = C^{2b1b}\widetilde{H}^{2b1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{68}$$

with

$$\begin{aligned}
C^{2b1b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)\\
\widetilde{H}^{2b1b}(m_q,u,v) &= -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]}\\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_3\cdot p_{J/\psi}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P\\
&-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V-64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{P}p_{J/\psi}\cdot p_V+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p'^{\nu}_3p_V\cdot p_P]\\
&+[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-96\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned}\tag{69}$$

where $p'_1=-p_3, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{2b2a}$, we have

$$\mathcal{A}^{2b2a} = C^{2b2a}\widetilde{H}^{2b2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{70}$$

with

$$\begin{aligned}
C^{2b2a} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{2b2a}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]}\\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_3\cdot p_{J/\psi}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P\\
&+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V+64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{P}p_{J/\psi}\cdot p_V-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p'^{\nu}_3p_V\cdot p_P]\\
&+[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned}\tag{71}$$

where $p'_1=-p_3, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{2b2b}$, we have

$$\mathcal{A}^{2b2b} = C^{2b2b}\widetilde{H}^{2b2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{72}$$

with

$$\begin{aligned}
C^{2b2b} &= \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
\widetilde{H}^{2b2b}(m_q,u,v) &= \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_6)^2[(p_4+p_6-p_2)^2-m_Q^2]}\\
& m_Qm_q\{D_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_3\cdot p_{J/\psi}-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_3\cdot p_P\\
&+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_3\cdot p_V-64\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_3p^{\beta}_{P}p_{J/\psi}\cdot p_V+64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p'^{\nu}_3p_V\cdot p_P]\\
&+[D_\theta(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(96\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P}-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+D^{\nu}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V\\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\}
\end{aligned}\tag{73}$$

where $p'_1=-p_3, p'_3=p_4+p_6-p_2$.

For amplitude $\mathcal{A}^{2c1a}$, we have

$$\mathcal{A}^{2c1a} = C^{2c1a}\widetilde{H}^{2c1a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{74}$$

with

$$C^{2c1a} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)$$
$$\widetilde{H}^{2c1a}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}$$
$$m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_P - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_V$$
$$-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_P - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_V$$
$$+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_P + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_P - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_V$$
$$-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_V - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_P\cdot p_V - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_P\cdot p_V]$$
$$+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})$$
$$+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\} \tag{75}$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_6$.

For amplitude $\mathcal{A}^{2c1b}$, we have

$$\mathcal{A}^{2c1b} = C^{2c1b}\widetilde{H}^{2c1b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{76}$$

with

$$C^{2c1b} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^aT^bT^c)$$
$$\widetilde{H}^{2c1b}(m_q,u,v) = \frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}$$
$$m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_P + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_V$$
$$-32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_P + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_V$$
$$+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_P + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_P - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_V$$
$$-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_V + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_P\cdot p_V + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_P\cdot p_V]$$
$$+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})$$
$$+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\} \tag{77}$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_6$.

For amplitude $\mathcal{A}^{2c2a}$, we have

$$\mathcal{A}^{2c2a} = C^{2c2a}\widetilde{H}^{2c2a}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{78}$$

with

$$C^{2c2a} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)$$
$$\widetilde{H}^{2c2a}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}$$
$$m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_P - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_V$$
$$+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_P - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_V$$
$$-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_P - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_P - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_V$$
$$-32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_V - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_P\cdot p_V - 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_P\cdot p_V]$$
$$+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})$$
$$+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_P + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_V + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_V\cdot p_P)]\} \tag{79}$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_6$.

For amplitude $\mathcal{A}^{2c2b}$, we have

$$\mathcal{A}^{2c2b} \;=\; C^{2c2b}\widetilde{H}^{2c2b}(m_q,u,v)\Phi_{J/\psi}\Phi_V\Phi_P \tag{80}$$

with

$$\begin{aligned}
&C^{2c2b} = \mathrm{Tr}(T^aT^bT^c)\mathrm{Tr}(T^bT^aT^c)\\
&\widetilde{H}^{2c2b}(m_q,u,v) = -\frac{i\pi^2}{(2\pi)^4}g_s^6(\frac{1}{4N_C})^3\frac{1}{(p_4+p_5)^2}\\
&m_Qm_q\{E_0(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}[32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_2\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_2\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_2\cdot p_{V}\\
&+32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p'_1\cdot p_{J/\psi} + 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p'_1\cdot p_{P} - 32\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p'_1\cdot p_{V}\\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{V}p_{J/\psi}\cdot p_{P} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_2p^{\beta}_{P}p_{J/\psi}\cdot p_{V}\\
&+32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\alpha}_{V}p'^{\mu}_1p^{\beta}_{P}p_{J/\psi}\cdot p_{V} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_2p^{\alpha}_{J/\psi}p_{P}\cdot p_{V} + 32\varepsilon^{\nu}_{J/\psi}\varepsilon^{*\beta}_{V}p'^{\mu}_1p^{\alpha}_{J/\psi}p_{P}\cdot p_{V}]\\
&+[E_{1\theta}(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\nu}_{V}p^{\alpha}_{P}p^{\beta}_{V}p^{\theta}_{J/\psi} + 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{V}p^{\theta}_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\nu}_{J/\psi}p^{\beta}_{P}p^{\theta}_{V})\\
&+E^{\nu}_1(m_q,u,v)\varepsilon_{\mu\nu\alpha\beta}(-64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{V}p_{J/\psi}\cdot p_{P} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\alpha}_{V}p^{\beta}_{P}p_{J/\psi}\cdot p_{V} - 64\varepsilon^{\mu}_{J/\psi}\varepsilon^{*\beta}_{V}p^{\alpha}_{J/\psi}p_{V}\cdot p_{P})]\}
\end{aligned} \tag{81}$$

where $p'_1 = p_1, p'_2 = p_1 - p_4 - p_5, p'_4 = p_6$.

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